

GCSE Foundation

Algebra KS4 Part 1 F

Solving linear equations

$$6x - 5 = 2x + 15$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

Subtract 2x from both sides

$$4x - 5 = 15$$

$$\begin{array}{r} +5 \\ +5 \end{array}$$

Add 5 to both sides

$$4x = 20$$

$$x = 5$$

Divide both sides by 4

Factorising

Factorise $x^2 + x - 42$

Sum of (+) 1
-6 + 7 = 1

product of -42

-6 x 7 = -42

$$x^2 + x - 42 = (x - 6)(x + 7)$$

Expanding Brackets

$$(5x - 3)(2x + 1)$$

x	5x	-3
2x	10x ²	-6x
+1	+5x	-3

$$= 10x^2 - 6x + 5x - 3$$

$$= 10x^2 - x - 3$$

Don't forget to simplify
-6x + 5x = -x

Rearranging formulae

$$y = ax + c$$

$$ax + c = y$$

$$ax = y - c$$

$$x = \frac{y - c}{a}$$

The 'aim' is to get 'x' on its own (x = ...)

Algebra – Keywords.

- Substitution – replacing a letter with a number. (Letters next to each other means to multiply.)

Eg. Work out $2g + 3h$ when $g=9$ and $h=4$

$$2 \times 9 + 3 \times 4$$

$$18 + 12$$

$$\underline{30}$$

- Expression – an algebraic sentence without an equal sign. (You may need to simplify but not solve.)

- Simplify – to make an expression have less terms.

Eg. $2a + b + 3a + 2b - a - 2b = 4a + b$

Eg. $5 \times r \times 2 \times p = 10rp$

- Solve – to work out an answer using algebra, to get $x = \dots$

- Term – one part of an expression separated by a + or -

Eg. $6p + 5q + 2r^2$ — different terms.

- Expand – multiply to get rid of brackets.

Eg. $3(x + 2) = 3x + 6$

- Factorise – opposite of expand, divide and put in brackets.

Eg. $3x + 6 = 3(x + 2)$
 $4x + 8 = 4(x + 2)$

- * Indices an algebraic term that has a power Eg. $3t^6$

- * Sequence – an algebraic pattern going up by the same amount each time. In year 8 work out the formula called the nth term

- * 'make x the subject' – re-arrange formula until it is in the form $x =$

Algebra KS4 Part 2 F

Inequalities

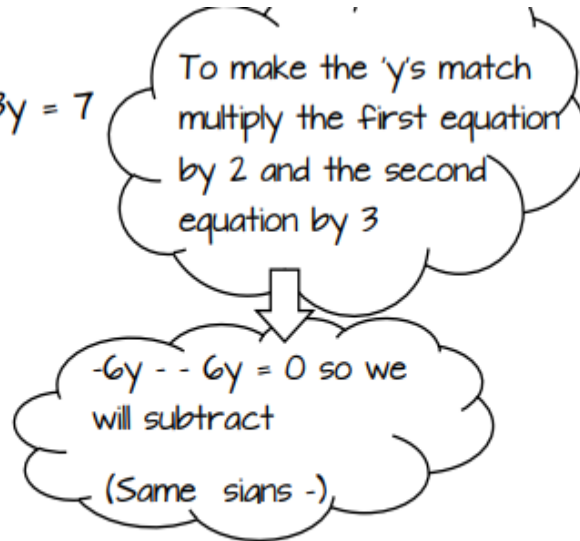
$$\begin{aligned}
 7x + 3 &< 2x + 18 \\
 5x + 3 &< 18 & (-2x) \\
 5x &< 15 & (-3) \\
 x &< 3 & (+5)
 \end{aligned}$$

SIMULTANEOUS EQUATIONS

 Solve simultaneously
 $3x - 2y = 8$ and $2x - 3y = 7$

$$\begin{aligned}
 9x - 6y &= 24 \\
 \underline{4x - 6y} &= 14 \\
 5x + 0 &= 10
 \end{aligned}$$

$$\begin{aligned}
 x = 2 \quad 6 - 2y &= 8 \\
 y &= -1
 \end{aligned}$$



We could have solved by matching the 'x' by multiplying the first equation by 2 and the second equation by 3

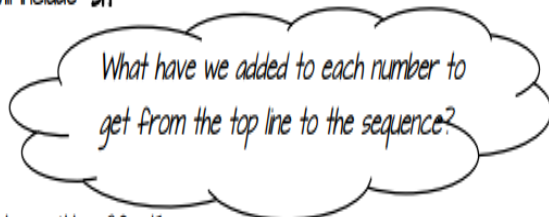
Sequences

Find the nth term: 15, 10, 5, 0

Look at the difference between consecutive terms - decreasing by 5 each time

So we know the nth term formula will include $-5n$

	-5	-10	-15	-20
Sequence	15	10	5	0



The nth term = $-5n + 20$ which can be written $20 - 5n$

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 $2 \times 9 + 3 \times 4$
 $18 + 12$
 30
- Expression – an algebraic sentence without an equal sign. (You may need to simplify but not solve.)
- Simplify – to make an expression have less terms.
 Eg. $2a + b + 3a + 2b - a - 2b = 4a + b$
 Eg. $5x \times r \times 2 \times p = 10rp$
- Solve – to work out an answer using algebra, to get $x = \dots$
- Term – one part of an expression separated by a + or -
 Eg. $6p + 5q + 2r^2$ → different terms.
- Expand – multiply to get rid of brackets.
 Eg. $3(x + 2) = 3x + 6$
- Factorise – opposite of expand, divide and put in brackets.
 Eg. $3x + 6 = 3(x + 2)$
- * Indices an algebraic term that has a power Eg. $3t^2$
- * Sequence – an algebraic pattern going up by the same amount each time. In year 8 work out the formula called the nth term
- * 'make x the subject' – re-arrange formula until it is in the form $x =$
- * The equation of a straight line in algebra is $y = mx + c$. Where m is the gradient (steepness) and c is where the line crosses the y axis
- * Simultaneous Equation – this is where you have 2 equations that you solve at the same time, getting values for x and y

Calculation KS4 Part 1 F

Ratio

Divide £48 in the ratio 3 : 5

3 : 5 8 parts in the ratio (3 + 5)

£48 ÷ 8 = £6 1 part = £6

3 parts = 3 × £6 = £18

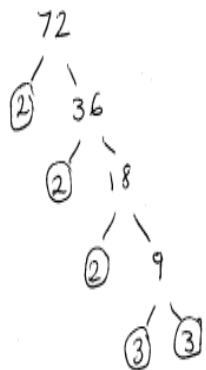
5 parts = 5 × £6 = £30

£18, £30

Prime Factor

Form

Write 72 as a product of its prime factors.



$$2 \times 2 \times 2 \times 3 \times 3$$

$$\text{or } 2^3 \times 3^2$$

$$2^3 \times 3^2$$

Calculation – Key Words

- Integer - a whole Number
- Fraction - a part of a number has a numerator on the top and a denominator on the bottom
- Equivalent Fraction - two fractions which have the same value but are written differently-
 $\frac{1}{2} = \frac{4}{8}$
- Percent - means out of 100, symbol %
- Multiple - any number in your original times table
- Factor - a number that goes into another number with no remainder
- Highest Common Factor - the biggest number that goes into two numbers - HCF of 12 and 16 is 4
- Lowest Common Multiple - the first number that appears in the times table of 2 different numbers - LCM of 3 and 5 is 15
- Prime Number - a number with only 2 factors, itself and 1
- Square number - the answer to a number multiplied by itself
 $1 \times 1 = 1$ $2 \times 2 = 4$ $3 \times 3 = 9$
- Cube Number - the answer to a number multiplied by itself twice
 $1 \times 1 \times 1 = 1$ $2 \times 2 \times 2 = 8$ $3 \times 3 \times 3 = 27$
- Square Root - Opposite of square number. This is the answer to what number multiplied by itself is the square number - $\sqrt{16} = 4 \times 4$ so square root of 16 is 4
- Product means to multiply
- Sum means to add
- Share means to divide
- Difference means to subtract
- Evaluate - work out the answer
- Ratio - is comparing one quantity against another, written as a : b
- Significant figures - Is rounding to the most important (biggest value) digit - 2567 to 1 sf is 3000
- Standard Form - Is a method of writing very large or very small numbers - $a \times 10^n$. Where a is bigger than 1 and smaller than 10
- BIDMAS - Gives the order in which a calculation should be done
- Recurring - A decimal that continues forever with the same number after the decimal point
- Prime Factor - A factor of a number that is also a Prime number.
- Mixed Fraction - A fraction that has a whole number and a fraction
- Top heavy Fraction - A fraction where the numerator is bigger than the denominator

Percentages

Decrease £350 by 10%

10% of £350 = £35 (divide by 10)

Decrease by 10% so take the 10% away from the original amount

£350 + £35 = £315

Fractions

$$\frac{1}{2} \times \frac{2}{3}$$

$$= \frac{5}{2} \times \frac{5}{3}$$

$$= \frac{25}{6} = 4\frac{1}{6}$$

Change to a mixed number

145 × 23 = Multiplication

$$\begin{array}{r} 145 \\ \times 23 \\ \hline 435 \\ 2900 \\ \hline 3335 \end{array}$$

Start with working out $3 \times 145 = 435$

Now work out 20×145 don't forget the zero

$$145 \times 23 =$$

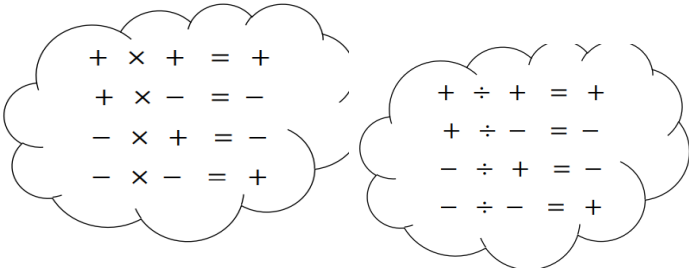
	100	40	5
20	2000	800	100
3	300	120	15

$$2000 + 800 + 100 +$$

$$300 + 120 + 15 = 3335$$

Calculation KS4 Part 2 F

Directed Numbers



$$-4 \times 3 = -12$$

$$-24 \div -3 = 8$$

$$-4 \times -3 = 12$$

$$4 \times -3 = -12$$

$$4 \times -3 = -12$$

INDICES

(a) Simplify $x^8 \times x^3$

(b) Simplify $(5y)^3$

$$5y \times 5y \times 5y$$

(c) Simplify $\frac{w^7}{w^4}$

$$\frac{x^8 \times x^3}{(1)}$$

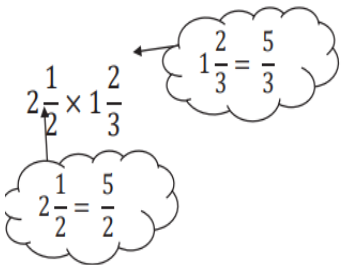
$$\frac{125y^3}{(1)}$$

$$\frac{w^3}{w}$$

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- Mixed Fraction - A fraction that has a whole number and a fraction
- Top heavy Fraction – A fraction where the numerator is bigger than the denominator
- Indices – are another name for powers. There are rules with indices

Fractions



$$2\frac{1}{2} \div 1\frac{2}{3}$$

$$= \frac{5}{2} \div \frac{5}{3}$$

$$= \frac{5}{2} \times \frac{3}{5}$$

$$= \frac{15}{10} = \frac{3}{2} = 1\frac{1}{2}$$

Standard Form

Work out $6 \times 10^3 \times 5 \times 10^2$

$$6 \times 10^3 \times 5 \times 10^2$$

$$= 6 \times 5 \times 10^3 \times 10^2$$

$$= 30 \times 10^5$$

$$= 3 \times 10^6$$

Make sure your answer is in standard form

$$10^a \times 10^b = 10^{a+b}$$

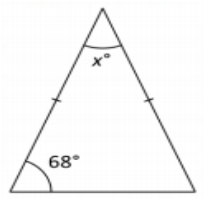
Geometry KS4 Part 1 F

$$x^\circ + 68^\circ + 68^\circ = 180^\circ$$

$$x^\circ + 136^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 136^\circ$$

$$x^\circ = 44^\circ$$



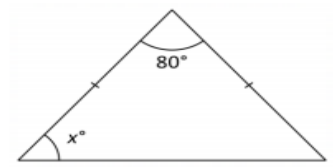
In an isosceles triangle 2 of the angles are the same size

$$x^\circ + x^\circ + 80^\circ = 180^\circ$$

$$2x^\circ + 80^\circ = 180^\circ$$

$$2x^\circ = 100^\circ$$

$$x^\circ = 50^\circ$$



Parallel lines

Alternate angles are equal

Corresponding angles are equal

Allied (or co-Interior) angles add up to 180

Polygons

$a + b + c + d + e + f = 360$

Exterior angles of a polygon add up to 360

$a + b = 180$

The Interior and exterior angle of any polygon add up to 180

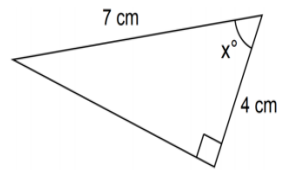
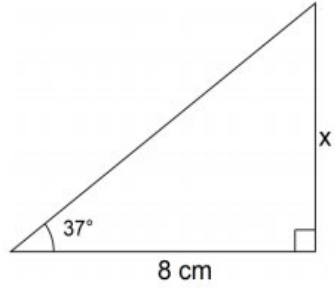
Geometry Key Words and Formula's

- * Area – the space inside a 2D shape measured in units 2
- * Perimeter - the distance around the outside of a shape (called circumference for circles)
- * Volume – the space inside a 3D shape
- * Surface Area – the area of the flat faces of a 3D shape
- * Angle - The space made when two lines meet, measured in degrees
- * Acute angle – less than 90° , obtuse angle bigger than 90° smaller than 180° . Straight line angle equal to 180° . Reflex angle bigger than 180° but smaller than 360° .
- * Angles in a straight line add to 180°
- * Angles in a triangle add to 180°
- * Angles around a point add to 360°
- * Parallel lines – these lines have the same gradient and they never meet
- * Perpendicular lines – these lines cross at 90°
- * Alternate angles – these two angles are the same in parallel lines (Z angle)
- * Corresponding angles – these two angles are the same in parallel lines (F angles)
- * Co – Interior angles – these two angles add up to 180° (C angles)
- * Scalene triangle – A triangle with three different sides and three different angles
- * Isosceles triangles – A triangle that has the two sides the same length and the base angles the same
- * Equilateral triangle – A triangle that has three sides the same and three angles the same
- * Polygon - A 2D shape that has only straight sides (edges)
- * Interior and Exterior angles – The exterior angles of any polygon always add to 360° . The interior angles + exterior angles always add to 180°
- * Bearing - A bearing is an angle that starts from North, goes **clockwise** and must have three figures in it
- * Reflection – is flipping a shape over a mirror line
- * Rotation – is spinning a shape from a certain point, the direction of movement is required
- * Translation – is pushing a shape horizontally and vertically. It is written as a vector
- * Enlargement – is making a shape bigger or smaller by a scale factor from a centre point

Geometry KS4 Part 2 F

Calculate x
 $\tan 37^\circ = \frac{x}{8}$

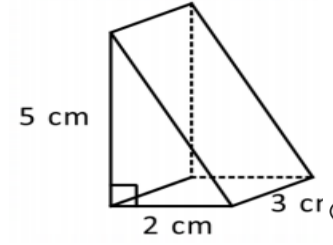
$x = 8 \times \tan 37$
 $x = 6.0 \text{ cm (1 dp)}$



Calculate x
 $\cos^\circ = \frac{4}{7}$

$x = \cos^{-1}(4 \div 7)$
 $x = 55^\circ$

Calculate the volume
 Area of the cross section (triangle) = $\frac{1}{2} \times 2 \times 5 = 5 \text{ cm}^2$



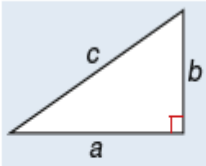
Volume = $5 \times 3 = 15 \text{ cm}^3$

Volume of a prism
 Area of the cross section \times length

Pythagoras

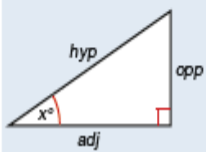
Pythagoras' Theorem

For a right-angled triangle,
 $a^2 + b^2 = c^2$



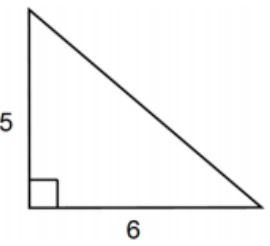
Trigonometric ratios (new to F)

$\sin x^\circ = \frac{\text{opp}}{\text{hyp}}$, $\cos x^\circ = \frac{\text{adj}}{\text{hyp}}$, $\tan x^\circ = \frac{\text{opp}}{\text{adj}}$



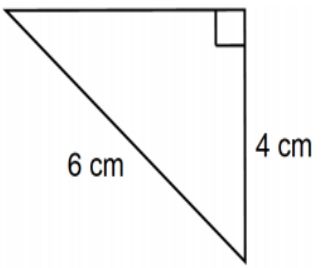
Calculate the length of the missing side

$5^2 + 6^2 = x^2$
 $x^2 = 61$
 $x = 7.8 \text{ cm (1 dp)}$



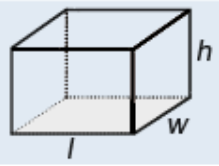
Calculate the length of the missing side

$x^2 + 4^2 = 6^2$
 $x^2 + 16 = 36$
 $x^2 = 20$
 $x = 4.5 \text{ cm (1 dp)}$

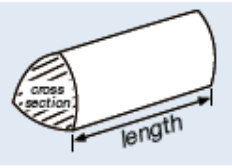


Volumes

Cuboid = $l \times w \times h$



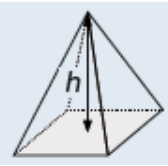
Prism = area of cross section \times length



Cylinder = $\pi r^2 h$



Volume of pyramid = $\frac{1}{3} \times \text{area of base} \times h$



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- * Translation – is pushing a shape horizontally and vertically. It is written as a vector
- * Enlargement – is making a shape bigger or smaller by a scale factor from a centre point
- * Net – is the 2D flat drawing of the 3D shape
- * Edges, Vertices and Faces – These are used to describe the parts of a 3D shape. The face is the flat side. The vertices are the corners, and the edges are the lines that join the corners
- * Prism – is a 3D shape that has the same cross-sectional area all the way through it
- * Congruent Shapes – Are shapes which are the same - they have the same angles and the same sides
- * Similar Shapes – Have the same angles but the sides are in ratio. Find the scale factor of the big shape to the smaller one
- * Pythagoras – is used for right angled triangles to find the length of a missing side. Square both, add or subtract then square root
- * SOH/CAH/TOA – is used to find either a missing side or a missing angle in a right-angled triangle
- * hypotenuse – is the longest side in a right-angled triangle
- * O, A H - are the opposite, adjacent and hypotenuse. Used with SOH/CAH/TOA

Statistics KS4 Part 1 F

AVERAGES

The **MODE** is the value that occurs most often

② 4, ② 6, 1, 4, ②

There are more 2's in the data list than any other number so the **MODE** = 2

The **MEDIAN** is the value in the middle when the data list listed in order

Example 1

2, 5, 4, 2, 1

Write the data in order

1, 2, 2, 4, 5

MEDIAN = 2

Example 2

4, 5, ⑤, ⑧, 10

If there are 2 numbers in the middle

$$\text{MEDIAN} = \frac{5+8}{2} = \frac{13}{2} = 6.5$$

To calculate the **MEAN**

3, 6, 3, 10, 8

Step 1: Add all of the 'numbers' together

$$3 + 6 + 3 + 10 + 8 = 30$$

Step 2: Divide your total by the number of values

$$\begin{aligned} \text{in your data list} \quad \text{MEAN} &= 30 \div 5 \\ &= 6 \end{aligned}$$

The **RANGE** is the difference between the largest and smallest number in the list $10 - 3 = 7$

Statistics Key Words

Range – the difference between the largest and the smallest number from a list of numbers

Mode – The number that appears the most often from a list of numbers

Median – the number in the middle of a list of ordered numbers

Mean – Add all the numbers up and then divide this total by the amount of numbers that were there

Averages – Mean, Median and Mode. Three averages to help determine common or a representative number from a list of numbers

Pie Charts – A way of representing data in a circle. All pie charts add up to 360°.

Probability - the chance of an event happening. Probability must be written as a fraction, decimal or a percentage. Not as a ratio.

Tree Diagrams – A diagram to show the probability of two or more events happening

Probability 'OR' Rule – The Probability of event A or event B happening is $P(A) + P(B)$

Probability 'AND' Rule – The Probability of event A and event happening is $P(A) \times P(B)$

Probability – of all possible events adds to 1

Venn Diagrams – Uses two circles often overlapping to show data

Scatter Diagram – A graph that shows the relationship between two variables

Correlation – Used to describe the relationship in scatter diagrams – positive both go up or down, negative – one goes up as the other goes down, no correlation – there is no link between the two variables

Line of best Fit – A straight line drawn through the scatter diagram with roughly half the data points on either side of the line

Frequency Table – Data is put into groups in a table. Used to help find averages

Pictogram – Is a way of showing data using pictures. It must have a key explaining what the picture stands for

Stem and Leaf – Shows numbers in a table, the leaf is the last digit of the piece of data. Needs a key

Pictogram

The pictogram shows information about the number of books sold by an author in January, February and March.

January	
February	
March	
April	

Key:	
	represents 8 books

(a) Write down the number of books sold in March

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} = 8 \quad \square = 2$$

14 books were sold in April.

(b) Show this information on the pictogram.

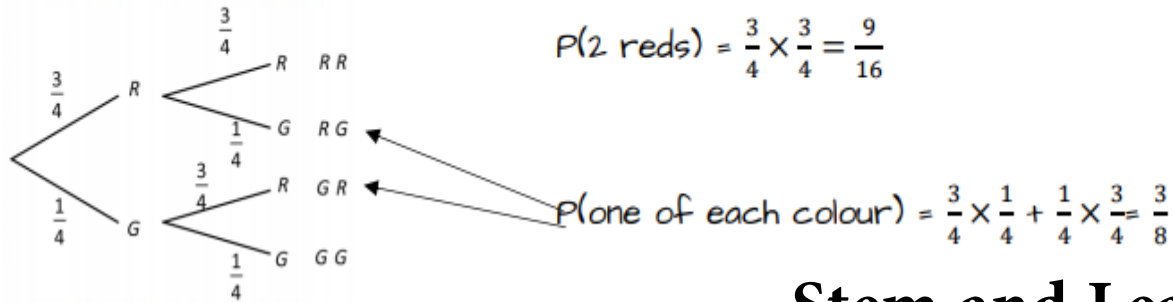
$$\begin{array}{r} 26 \\ \hline (1) \end{array}$$

(1)

Statistics KS4 Part 2 F

Probability

A bag contains 3 red and 1 green ball. A ball is picked at random, the colour noted and then replaced before second ball is picked



Stem and Leaf

Mean from a table

Adam is measuring the heights in cm of his tomato plants.

Height (cm)	m.p		Frequency
140 < h ≤ 150	145	x	7
150 < h ≤ 160	155	x	10
160 < h ≤ 170	165	x	15
170 < h ≤ 180	175	x	19
180 < h ≤ 200	190	x	9

m.p x f

1015

1550

2475

3325

1710

10075

60

Here are the heights, in cm, of 18 people.

~~180~~ ~~168~~ ~~186~~ ~~186~~ ~~158~~ ~~190~~ ~~165~~ ~~184~~ ~~188~~
~~183~~ ~~184~~ ~~189~~ ~~183~~ ~~183~~ ~~159~~ ~~167~~ ~~177~~ ~~162~~

Draw an ordered stem and leaf diagram to show this information.

15	4 3 4 8 9
16	2 5 7 8
17	7
18	3 3 4 5 6 6 9
19	0 0

Key: 15|3 = 153cm

(a) Estimate the mean height.
Give your answer correct to 1 decimal place.

$$\frac{10075}{60} = 167.9 \text{ (1dp)}$$

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